Telecommunications and Radio Engineering, 74 (12):1039-1050 (2015)

MICROWAVE ELECTROMAGNETICS

GREEN'S FUNCTION FOR AN INFINITE ANISOTROPIC MEDIUM. REVIEW

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A review of basic papers related to development and implementation of rather efficient mathematical methods aimed at formation of Green's functions for various types of anisotropic media: uniaxial and biaxial, generalized gyrotropic, layered etc. is provided. Advantages and drawbacks of the methods aimed at both the analytical results and the possibility of numeric or asymptotic estimates of the sought functions are highlighted. The outlook for using them while solving practical problems of current importance are determined.

KEY WORDS*: anisotropic medium, dyadic Green's function, explicit representations, asymptotic representations, numerical analysis, Fourier transform*

1. INTRODUCTION

Studying of particularities of the processes of radio waves propagation and scattering in anisotropic media is important for various spheres of science like optics and physics of crystals, physics of plasma and theory of composites, geophysics, antenna theory and engineering and remote sensing. The most general approaches to description of the processes of radio waves propagation and scattering in anisotropic media can be found in the treatises [1-4], while Green's function [5] is the most efficient tool used for analysis of such processes. The dyadic Green's function (DGF) for an unbounded anisotropic medium is usually represented by the 3D Fourier integral in the space of the wave vector $\vec{k}$. It is a standard procedure. The problem itself is in computing of the said integral.

Exact closed-form representation of Green's function in a dyadic, independent upon coordinates, form is obtained for a uniaxial infinite anisotropic medium of both electric and magnetic types [2,6], as well as for the medium having both types of the above anisotropy simultaneously [7]. However such, a closed form is apparently impossible for more general anisotropic media [8-11]. DGF represented by the Fourier integral from a non-decreasing at infinity function is a generalized function. Therefore, it is reasonable to solve the problem of its formation separately for its qualitatively different parts – the singular (generalized function) and the regular (normal function).

The present review of basic papers related to analysis of Green's functions in various infinite anisotropic media is divided in three parts considering the state-of-the-art of the problem of formation of stationary and non-stationary Green's functions for homogeneous and layered anisotropic media. Complete list of the papers worth mentioning in a review like this must be, of course, much longer. Nevertheless, we had to limit ourselves to the provided one hoping that everyone to whom it might concern could find in the reference literature and quotes from the books and papers referenced to in this paper [1-54].

2. DYADIC GREEN'S FUNCTION FOR A HOMOGENEOUS ANISOTROPIC MEDIUM

2.1 Closed-Form Dyadic Green's Functions

It is well-known a closed-form of DGF for a uniaxial dielectric medium [2,12]. A general uniaxial medium (the permittivity and permeability tensors are $\underline{\overline{\varepsilon}} = \varepsilon_1 \underline{\overline{I}} + (\varepsilon - \varepsilon_1)\vec{c}\vec{c}$ and $\underline{\overline{\mu}} = \mu_1 \underline{\overline{I}} + (\mu - \mu_1)\vec{c}\vec{c}$; $\underline{\overline{I}}$ is the unit tensor; $\vec{c}$ is the unit vector parallel to the separated anisotropy axis; ε , ε_1, μ , μ_1 are constant values) is considered in the paper [7]. All of the four dyadic functions (of electric, magnetic and mixed types) are provided and properties of their symmetry are analyzed.

The paper [13] contains the methods, the efficiency of which while developing fundamental solutions (Green's functions) to differential operators of the electromagnetic theory, is proved quite convincingly. It is provided a complete list of the media, for which DGF are obtained in a closed form. The list includes isotropic and bi-isotropic (including also optically active ones) media; generally uniaxial and electrically or magnetically gyrotropic media; diffusive and moving media; along with some kinds of plasma. The paper critically evaluates the achievements, applicability limits, failures and perspectives of the analytical methods – if a medium under consideration appears to be more complexly built than the gyrotropic one or an inhomogeneous, then properly analytical methods turn out to be practically inadequate.

2.2 Asymptotic Representations

At the very beginning, general consideration of the problems of electromagnetic radiation in a magneto-ionic medium was stimulated by the requirements set by artificial earth satellites and rockets in the ionosphere. Bunkin F.V. determined the expression in the form of a Fourier integral for the current radiation field in electrically anisotropic arbitrary medium and obtained a precise expression for the Hertz vector in the far-field zone for the case of a gyro-electric medium, and determined the electric dipole electric field [14] in the same zone. Using a Fourier transform, Kogelnik H. estimated the electric dipole radiation power in the medium concerned [15]. Detailed review of the papers in this trend is provided in the thesis [16]. It has to be mentioned that it is just the paper [14] that played an important role in development of the theory of current source radiation in an anisotropic media. Even the mistake made by the author – incorrectly determined contribution of multiple stationary points while performing asymptotic estimation of the fields in far-field zone – has not impaired that.

Already before the closed-form expression for the DGF of a uniaxial anisotropic medium was obtained, in order to estimate the relevant Fourier integrals, Chen H.C. [17] had applied the method previously developed by Lighthill M.J. The method distinctive feature was in the fact that with its help it was easy to track the field behavior in the far-field zone at variation of the shape of the dispersive surface.

An analytical method that allows obtaining asymptotes of DGF in the far-field zone directly upon its known Fourier transform without using an inverse transform is provided for and electrically uniaxial anisotropic medium in the paper [18]. The basis of the method is formed by applying the duality principle between the wave and beam vectors. The authors assume that a similar approach would work efficiently in the case of a biaxial anisotropy as well; however they provide no explanation for the grounds of the above assumption.

The asymptotic approach is developed the most substantially in the paper [19]. In that paper studying of DGF in the far-field zone of a biaxial anisotropic medium is based upon the ideas by Lax M. and Nelson D.F. [20,21]. A complex structure of the wave and the beam surfaces and their singular points are described. As the result of a detailed analysis of the Gaussian curvature of the wave surface, there were determined the directions, in which asymptotic behavior of DGF was different from the normal one – $O\left(r^{-1}\right)$, where r is the distance to the observation point. These directions are related to the biradials of a biaxial medium (here DGF behaves as $O\left(r^{-1/2}\right)$) and to the directions of the wave vector along each of the binormals (here DGF behaves as $O\left(r^{-5/4}\right)$). Smooth transition of the DGF behavior correspondent to the value $O\left(r^{-1/2}\right)$ to the normal behavior correspondent to the value $O\left(r^{-1}\right)$ is analyzed. A possibility of experimental observation of a non-standard asymptotic behavior of DGF is discussed.

2.3 Green's Functions For Complex Anisotropic Media. Coordinate-Free Approach

The paper [22] considered a generalized gyrotropic medium – the anisotropic medium with a separated axis and electromagnetic parameters with a rotational invariance with regards to that axis. It is a model of cold ionosphere plasma in the Earth's magnetic field. It is shown how the DGF-tensors can be represented in terms of two Hertz scalar potentials.

An infinite homogeneous medium with a general anisotropy (all nine components of the permittivity and permeability dyads $\underline{\overline{\varepsilon}}$ and $\underline{\overline{\mu}}$ are different from zero) is investigated in the paper [23]. Coordinate-free representations of all four spatial DGF $\underline{\overline{G}}_{\nu\xi}\left(\vec{r}-\vec{r}'\right)$ are determined in terms of one scalar potential $W\left(\vec{r}-\vec{r}'\right)$. Exact explicit expressions are obtained for the Fourier transform $\underline{\overline{G}}_{\nu\xi}\left(\vec{k}\right)$ of the above DGF. On the basis of the above it is formed the asymptotic expression for a scattered field of an arbitrary source in the far zone upon any of possible directions including the directions related to singular optic axes. Unfortunately, these results are not matched with similar results from the paper [19]. In [23] it is also described the static part of DGF, which is used subsequently at a rigorous description of the field inside of a small homogeneous anisotropic ellipsoid in the anisotropic environment. The effects of wave scattering upon the said ellipsoid are estimated in a quasi-static approximation.

New formalism of DGF formation using computation of differential forms is suggested in the papers by Warnick K.F. et al. [24-27]. DGF for an inhomogeneous anisotropic medium is represented as a double differential form. For the general case, material properties as characterized by the permittivity and permeability tensors are embedded into the Hodge star operator. The authors are of the opinion that such formalism is essentially simplifying the work with differential operators inherent to an inhomogeneous anisotropic medium. One of the specific results obtained by Warnick K.F., is related to development of a formation scheme the for DGF for a homogeneous biaxial medium as a solution to an integral equation.

2.4 Green's Functions for Complex Anisotropic Media. Coordinate Approach

The coordinate approach using the Cartesian, cylindrical or spherical system of coordinates in the space K^3 of the wave vectors $\vec{k}$ is prevailing at analyzing the Fourier integral for DGF in the case with biaxial and more complex anisotropic media. Considering that it is impossible to reduce the integral to an explicit closed-form, most papers in this direction of research are related to its transforming into the form allowing numerical analysis or asymptotic estimations of DGF in the near- and far-field zones.

Radiation of an elementary current source placed into an infinite biaxial anisotropic medium is investigated analytically in the paper [8]. Using the Fourier analysis it is demonstrated that the radiation field can be represented in the form of spherical waves divided into "standard" and "non-standard" waves. It is proved that the field has the order $O\left(r^{-3}\right)$ in the vicinity of the source, and the far field is represented by two different spherical waves. The developed approach, at implementation of which integration upon a radial variable in the basic triple integral is replaced with double summation, appears too bulky for numerical estimations of the field in the arbitrary points of observation.

The papers [9,11] are related to studying of properties of the DGF for an infinite biaxial anisotropic medium. In the first of them, the triple Fourier integral for DGF considered in the Cartesian coordinates in K^3 is reduced using the method of deductions to a double integral in infinite limits. In the second paper in the same integral, which is considered in cylindrical coordinates, the integration is performed upon the longitudinal Fourier variable. It is separated the singular part of DGF, the form of which depends not only upon the selected system of coordinates but also upon the order of integration upon the longitudinal and radial Fourier variables (the so-called pillbox- and a needle-shaped principal volume).

Using the method suggested in [11], DGF for a more general anisotropic medium characterized with the following relative permeability tensor, is analyzed in the paper [28]

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \quad \varepsilon_{xx}, \varepsilon_{xy} \text{ and so on are positive values.}$$

This model allows considering various physically realizable media – plasma, biaxial crystals etc. There are separated and analyzed two possible representations of a singular part of DGF for such a generalized anisotropic case. It is demonstrated that at $\varepsilon_{xy} = \varepsilon_{yx} = 0$ the obtained results form a match with the results of the paper [11].

A new approach to description of a complete set of DGF for an infinite electrically gyrotropic medium is suggested in the paper [29]. Analysis of expressions for DGF of electric and magnetic types is performed in terms of characteristic wave fields for the first and the second types of waves existing in a gyrotropic medium. DGF for electro-magnetic and magneto-electric types $\underline{\overline{G}}^e_{em}(\vec{r},\vec{r}\,')$ and $\underline{\overline{G}}^e_{me}(\vec{r},\vec{r}\,')$ are expressed via DGF $\underline{\overline{G}}^e_{mm}(\vec{r},\vec{r}\,')$ и $\overline{\overline{G}}^e_{ee}(\vec{r},\vec{r}\,')$ correspondingly. It is shown that dyadic decomposition is essentially simplifying calculation of the inverse operator. A simplified option of the above approach is provided in [30]. It is based on representation of the fields in the space K^3 in terms of the wave matrices. Fourier transform of DGF is reduced to inversion of the wave matrices. The duality relationships for DGF are described in detail. The anisotropy type considered in [29,30] is a particular case of the anisotropy type considered in the paper [23]. However, there was not performed any matching of the implemented approach with the approach and results of the paper [23].

Theoretical analysis of the electromagnetic response from a homogeneous biaxial anisotropic medium characterized by a permittivity tensor is provided with regards to the problems of geophysics in the paper [31]. A rather full review of the papers in this aspect is provided.

The paper [32] includes description of formation of DGF for a general bi-anisotropic medium, the constitutive equations for which have the representations $\vec{D} = \overline{\underline{\varepsilon}} \cdot \vec{E} + \overline{\underline{\xi}} \cdot \vec{H}$ and $\vec{B} = \overline{\underline{\zeta}} \cdot \vec{E} + \overline{\underline{\mu}} \cdot \vec{H}$. Complexity of the above problem is stipulated by a large number of free parameters. In a general sense all the tensor elements $\overline{\underline{\varepsilon}}$, $\overline{\underline{\xi}}$, $\overline{\underline{\mu}}$ and $\overline{\underline{\zeta}}$ are non-zero ones. The authors obtained in an explicit form the DGF for an electric field in the spectral domain K^3 . It is sufficient for estimation of asymptotic behavior of the DGF in the space R^3 in near and far-field zones of the source. The paper offers a genuine approach to satisfying the radiation at infinity condition. The approach is based upon introducing a special spherical system of coordinates aimed at the direction to the point of observation, but its realization has been performed with errors in [32].

3. DYADIC GREEN'S FUNCTION FOR A STRATIFIED ANISOTROPIC MEDIUM

The necessity of detailed studying of wave processes in a stratified anisotropic medium is stipulated by numerous practical applications – from geophysical survey and remote sensing to electromagnetic simulation of microwave antennae and waveguides. There were suggested and applied various methods of formation of DGF for layered planar anisotropic structures related to the Fourier transform with different types of decomposition upon eigen functions and matrix formulations.

The papers [33,34] suggest the method for solving the problem of excitation of a layered medium with complex anisotropic properties, which is based on a double Fourier transform upon longitudinal with respect of the scattering structure coordinates. The problem is reduced to determining the spectral DGF. It is determined the $2n \times 2n$ DGF matrix (n is the number of layers; in [33] $n = 2$), using of which allows calculation with the help of the method of moments of main radiation characteristics of the sources positioned on the layers separation boundaries.

A three-layer structure consisting the layer possessing general electric and magnetic anisotropy and two adjacent to it isotropic half-spaces is considered in the paper [35]. Anisotropy axes have an arbitrary spatial orientation. The source is represented by an elementary electric dipole arbitrarily oriented inside of the isotropic half-space, or a plane electromagnetic wave incident upon the anisotropic layer. The Fourier transform method and the matrix analysis technique are applied. It is formed a radiation pattern for the point dipole propagated through the anisotropic layer of an anisotropic medium, and the expressions are obtained for generalized scattering matrices elements of the above layer.

An efficient method for formation of DGF for a multilayer symmetric gyroelectric medium ($\varepsilon_{xy} = \varepsilon_{yx}$) is developed in a number of papers by Barkeshli S. [36-39]. The method is based on the Lorentz reciprocity principle [36] and the method of multiple scattering. There are formed the so-called transition and transmission matrices determining the efficiency of the field transmission through the layer and from one layer to another, correspondingly. Total decomposition of all the four DGF is provided at availability of both electric and magnetic point currents upon eigen vector wave functions. Some of the most interesting properties of DGF are considered, and their physical interpretation is given. The possibility of the method generalization for the case with the Hermite matrices $\overline{\underline{\varepsilon}}$ is analyzed.

The DGF in the spectral domain for a three-layer structure including the layer of biaxial anisotropic dielectric with arbitrarily directed axes of anisotropy is obtained in the paper [40]. Examples of numerical realization of the results related to estimation of the reflective property of the structure are provided. It should be noted an important particularity of the paper [40]. It is in the fact that that the representations similar to those formed here in [41] appeared to be useful at studying and analysis of different problems of wave scattering by a uniaxial anisotropic random medium [42].

The integral transforms method and the asymptotic decomposition technique for estimation of the Sommerfeld-type integrals are applied in the paper [43] for the analysis of the field of a microstrip dipole positioned on a uniaxial anisotropic substrate. The infinite double integral for an impedance matrix of the above structure is reduced to a finite unidimensional integral. This integral can be easily estimated numerically after separation and analysis of its singular part. Efficiency and accuracy of the implemented approach are demonstrated.

Total decomposition of DGF for an arbitrary planar anisotropic medium upon eigen functions and using the technique of cylindrical vector wave functions is represented in the paper [44]. General DGF is formed for a multilayered anisotropic medium. The approach details are illustrated by obtaining a full set of coefficients for the DGF for a four-layered medium. The results are used for studying radio waves propagation through the forest mass – a thin anisotropic layer. Field distribution of the electric dipole positioned in such a layer is obtained in the integral form.

In the paper [45] the far field radiated by an arbitrarily oriented Hertz dipole positioned inside or outside of the layer of a uniaxial anisotropic dielectric, the optical axis of which is oriented randomly, is investigated analytically using the DGF method. Asymptotic estimates obtained with the help of the fastest descent method are provided. Radiation field dependences upon basic parameters of the model – anisotropy mode, layer thickness and dipole position, are studied.

The matrix propagator method developed for solving electromagnetic problems of excitation by an arbitrary dipole source in stratified media, is disclosed in detail for a medium with general anisotropy and losses in the paper [46]. Reflection and transmission matrices (RT-response) are calculated at each media separation boundary, the recursive scheme for calculation of RT-matrices is provided for the entire layered structure, expressions for the vector field are obtained for each of the layers in the terms of eigen values and eigen vectors of the RT-matrices and functions of the source. The paper is designed for geophysical applications like subsurface sensing. In this connection it is considered a particular case of anisotropy related to the so-called transverse isotropy with a vertical axis (TIV) medium. For the case when the source and the receiver are in a TIV-medium, there are determined accurate expressions for the electromagnetic field of an arbitrary point dipole. In the case when the entire structure if formed by the layers possessing TIV-anisotropy, the recursive correlations for RT-responses are simplified to scalar equations. In that case the 2D Fourier transform is reduced to a 1D Hankel transform in this case.

In the paper [47] the recursive algorithm is adapted to calculation of DGF for a stratified uniaxial anisotropic medium with an arbitrary number of layers. Absence of subdivision of the field in the layers into the fields of outgoing and incoming waves is a particular feature of the implemented approach. Sequence of the systems of three linear equations with respect of the coefficients of the Sommerfeld integrals, originating at satisfying the conditions of field continuity at each boundary of the layers is inverted with the help of the recursive algorithm. The problem of singularity of the Sommerfeld integrals is solved, i.e., the integration loops are deformed in a proper manner within the complex plane. The representation obtained for DGF is compact and convenient for programing.

If the domain where the source is positioned, is filled with a reciprocal medium like, for example, a uniaxial medium, then to find the DGF it is possible, based on the properties of symmetry [42], to use a well-known representation of DGF for the case when the source is in the isotropic domain. However, the possibilities of application of the properties of symmetry are limited. First, they cannot be used to obtain DGF for the layered medium, if the limiting domain is filled with a non-reciprocal medium. Second, the properties of symmetry do not allow finding the DGF for a field in the domain where the source is positioned. The paper [48] is devoted to overcoming of the above restrictions. Here, there were first investigated DGF for infinite and layered medium with an arbitrary anisotropy using the method of decomposition upon eigen functions and the matrix method. Then, it was determined a complete set of DGF for all the domains of a layered structure with the source positioned in the isotropic domain. There were suggested modified properties of symmetry, with the help of which DGF was formed for the field both in isotropic and in anisotropic domains with the source positioned inside of one of anisotropic layers bordered by isotropic half-spaces.

Calculation of DGF is usually performed in two stages. First, it is analytically obtained its spectral representation. Then, using the inverse Fourier transform it is found a spatial representation of DGF reduced to the Sommerfeld integrals, which are not subject to analytical calculation. At availability of singularities in the vicinity of the integration loop along the entire real axis, numerical estimation of the above integrals from fast oscillating and slowly decreasing functions has to be obtained for every value of the frequency and all the possible sets of parameters of the source and point of observation. A number of investigations aimed at development of fast algorithms for calculation of DGF for a multilayered medium in the spatial domain have appeared during the recent decade. The paper [49] provides a detailed analysis of four of the above methods – the method of discrete complex images, the fastest descent method, the method of window-functions and the fast Hankel transform method. Comparison of the above methods in terms of accuracy and efficiency is performed, substantiated recommendations related to their practical application are provided, and a review of the papers related to the given subject is done.

The method of Green's functions is developed in [50] for calculation of electromagnetic fields generated by a source placed into a nanostructure medium of finite thickness described by an efficient electric permeability tensor. This approach is generalizing the method of decomposition of main modes into eigen modes of the medium and allows considering surface effects, in particular, the effect of modes transform at the boundary of an anisotropic medium. It is aimed at studying spatial distribution of the electromagnetic response of an artificial anisotropic medium, the properties of which can be described by an efficient permeability tensor.

The method for calculation of electromagnetic fields generated by arbitrarily oriented dipoles in a planar stratified medium, where each layer possesses an arbitrary independent anisotropy upon, generally speaking, both complex permittivity and permeability, is developed in the paper [51]. Analysis of 2D spectral integrals forms the basis of the above method. It is developed an efficient algorithm oriented both at interpreting of the geological survey results and at modeling of the field generated by the electric current in various stratified structures.

4. NON-STATIONARY GREEN'S FUNCTION

Propagation of the transient electromagnetic waves in a stratified anisotropic dispersive medium is considered in the paper [52]. The medium is modeled by a constitutive equation with the 3×3 matrix-valued susceptibility operator containing a time convolution integral. In the general case it is characterized by nine different susceptibility kernels. The problem of scattering of a plane wave, which is obliquely incident upon a finite-thickness layer containing a stratified anisotropic medium, is solved using the wave splitting method. The scattered field is represented by a time convolution of the incident field with the kernels of reflection and propagation. The above kernels are determined using two different methods – the method of submersion and the Green's function method. An exact analytical expression for the wave front of a secondary wave is provided for one specific class of media.

A new method for formation of Green's matrix function for a system of the non-stationary Maxwell equations for a homogeneous non-dispersive dielectric with a general form of anisotropy is described in the paper [53]. The method is implemented in two stages. First, it is determined the explicit form of the matrix function (Fourier image of Green's matrix function with regards to three spatial variables) depending upon the time variable and three Fourier parameters. At the second stage the determined matrix function is subject to the inverse Fourier transform. The efficiency of the above method is proved by the results of numerical experiments. A number of examples of electromagnetic fields modeling in anisotropic structures are considered.

Another new method for calculation of a fundamental solution to the equations describing transition processes in a general anisotropic non-dispersive medium is suggested in the paper [54]. It is divided in three basic stages – reduction of the matrix columns providing for a fundamental solution to symmetric hyperbolic systems;

formation of the Fourier images of those columns; and determining of a fundamental solution using an inverse Fourier transform. The method efficiency is demonstrated by the results of numeric experiments.

5. ACKNOWLEDGMENT

The work was carried out under the financial support of Ministry of Education and Science of the Republic of Kazakhstan.

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